RESEARCH ARTICLE

On Modification of Mismatch-Induced Stresses during Microwave Annealing of Radiation Defects Generated during the Radiation Processing of a Heterostructure

E.L. Pankratov[1,2]
[1]Nizhny Novgorod State University, 23 Gagarin Avenue, Nizhny Novgorod, 603950, Russia
2Nizhny Novgorod State Technical University, 24 Minin Street, Nizhny Novgorod, 603950, Russia

ABSTRACT: In this paper we formulate recommendations to decrease mismatch-induced stress in a heterostructure during microwave annealing of radiation defects, which were generated during radiation processing. Also we considered an analytical approach to analyze mass and heat transfer during the considered process. The approach gives a possibility to take into account changes in the parameters of processes in space and time, as well as the nonlinearity of the considered processes.

KEYWORDS: heterostructure; radiation processing; microwave annealing; mismatch-induced stresses; analytical approach for modeling

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INTRODUCTION

At the present time different heterostructures are widely used for the manufacturing of solid-state electronic devices [1-5]. The lattice constants of the heterostructure layers have different values. In this situation, mismatch induced stress existing in layers of heterostructures. Different methods could be used to decrease the stress. One of them is selection of materials with smaller difference between lattice constants. Another one is using buffer layer between main layers of heterostructure with intermediate value of their lattice constants. In the framework of this paper we consider an alternative approach to decrease the value of mismatch-induced stress. In this study, the approach influence of radiation processing of materials of heterostructure on value of mismatch induced stress is considered.

In this paper, we consider a two-layer heterostructure. The heterostructure consist of an epitaxial layer and a substrate (see Fig. 1). Let us also consider the radiation processing of the heterostructure with future microwave annealing of radiation defects, which were generated during the processing. An accompanying aim of this paper is introduction of an analytical approach for analysis mass and heat transfer. The approach should give a possibility to take into account changes in the parameters of processes in space and time, as well as the nonlinearity of the considered processes.

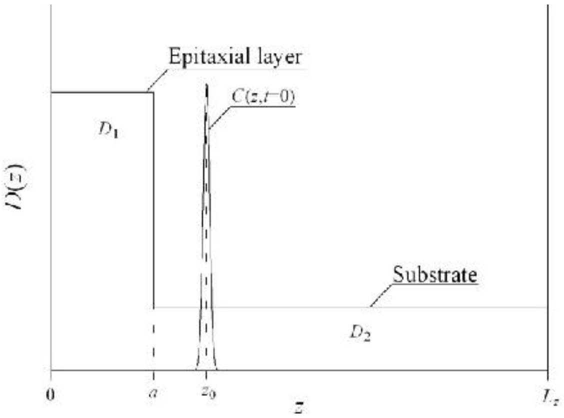

Fig. 1 Heterostructure containing an epitaxial layer and a substrate. This figure also shows the qualitative distribution of radiation defects in a semiconductor heterostructure

Method of Solution

In this paper the distributions of concentrations of radiation defects in space and time has been calculated by solution of the following system of equations [6-8]

$$\frac{\partial I(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial y}\right]+$$

$$+\frac{\partial}{\partial z}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial z}\right]-k_{I,V}(x,y,z,T)I(x,y,z,t)V(x,y,z,t)-$$

**Corresponding Author: elp2004@mail.ru*

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$$-k_{I,I}(x,y,z,T)I^2(x,y,z,t)+\Omega\frac{\partial}{\partial x}\left[\frac{D_{IS}}{kT}\nabla_S\mu(x,y,z,t)\int\limits_0^{L_z}I(x,y,W,t)dW\right]+$$

$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{IS}}{kT}\nabla_S\mu(x,y,z,t)\int\limits_0^{L_z}I(x,y,W,t)dW\right] \qquad (1)$$

$$\frac{\partial V(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial y}\right]+$$

$$+\frac{\partial}{\partial z}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial z}\right]-k_{I,V}(x,y,z,T)\,I(x,y,z,t)\,V(x,y,z,t)-$$

$$-k_{V,V}(x,y,z,T)V^2(x,y,z,t)+\Omega\frac{\partial}{\partial x}\left[\frac{D_{VS}}{kT}\nabla_S\mu(x,y,z,t)\int\limits_0^{L_z}V(x,y,W,t)dW\right]+$$

$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{VS}}{kT}\nabla_S\mu(x,y,z,t)\int\limits_0^{L_z}V(x,y,W,t)dW\right]$$

with initial and boundary conditions

$$\left.\frac{\partial I(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial y}\right|_{y=0}=0,$$

$$\left.\frac{\partial I(x,y,z,t)}{\partial y}\right|_{y=L_y}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial z}\right|_{z=L_z}=0,$$

$$\left.\frac{\partial V(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial y}\right|_{y=0}=0,$$

$$\left.\frac{\partial V(x,y,z,t)}{\partial y}\right|_{y=L_y}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial z}\right|_{z=L_z}=0,$$

$$I(x,y,z,0)=f_I(x,y,z),\ V(x,y,z,0)=f_V(x,y,z).$$

Here $T(x,y,z,t)$ is the spatio-temporal distribution of temperature; x, y and z are the spatial coordinated; t is time; ° $=I,V$; $I(x,y,z,t)$ and $V(x,y,z,t)$ are the spatio-temporal distributions of concentrations of interstitials and vacancies with equilibrium distributions I^* и V^*, respectively; functions $D_I(x,y,z,T)$, $D_V(x,y,z,T)$, $D_{IS}(x,y,z,T)$, $D_{VS}(x,y,z,T)$ describe dependences of volumetric and surficial (under influence of mismatch-induced stress) diffusion coefficients of point radiation defects defects on spatial coordinate and temperature; squares of concentration of point defects $I^2(x,y,z,t)$ and $V^2(x,y,z,t)$ gives a possibility to take into account generation of their simplest complexes with concentrations $c_I(x,y,z,t)$ and $c_V(x,y,z,t)$ (see, for example, Ref. [7]); function $k_{I,V}(x,y,z,T)$ describes dependence of parameter of point radiation defects recombination on spatial coordinate and temperature; the considered functions $k_{I,I}(x,y,z,T)$ and $k_{V,V}(x,y,z,T)$ describes dependences of the parameters of simplest complexes of point radiation defects generation on spatial coordinate and temperature; Ω is the atomic volume of dopant; ∇_s is the symbol of surficial gradient; $\int\limits_0^{L_z} C(x,y,z,t)\,dz$ is the surficial concentration of dopant on interface between layers of heterostructure (in this situation we assume, that Z-axis is perpendicular to interface between layers of heterostructure); k is the Biltzmann constant. Distributions of concentrations of radiation defects in space and time has been calculated by solution of the following system of equations [6-8]

$$\frac{\partial \Phi_I(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial y}\right]+$$

$$+\frac{\partial}{\partial z}\left[D_{\Phi I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial z}\right]+k_{I,I}(x,y,z,T)I^2(x,y,z,t)-k_I(x,y,z,T)I(x,y,z,t)+$$

$$+\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S « (x,y,z,t)\int\limits_0^{L_z}\Phi_I(x,y,W,t)\,dW\right]+$$

$$+\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S « (x,y,z,t)\int\limits_0^{L_z}\Phi_I(x,y,W,t)\,dW\right] \quad (6)$$

$$\frac{\partial \Phi_V(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial y}\right]+$$

$$+\frac{\partial}{\partial z}\left[D_{\Phi V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial z}\right]+k_{V,V}(x,y,z,T)V^2(x,y,z,t)-k_V(x,y,z,T)V(x,y,z,t)+$$

$$+\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S « (x,y,z,t)\int\limits_0^{L_z}\Phi_V(x,y,W,t)\,dW\right]+$$

$$+\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S « (x,y,z,t)\int\limits_0^{L_z}\Phi_V(x,y,W,t)\,dW\right]$$

with initial and boundary conditions

$$\left.\frac{\partial \Phi_\circ(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial \Phi_\circ(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial \Phi_\circ(x,y,z,t)}{\partial y}\right|_{y=0}=0,$$

$$\left.\frac{\partial \Phi_\circ(x,y,z,t)}{\partial y}\right|_{y=L_y}=0,$$

$$\left.\frac{\partial \Phi_\circ(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial \Phi_\circ(x,y,z,t)}{\partial z}\right|_{z=L_z}=0,\ c_I(x,y,z,0)=f_{cI}(x,y,z),\ c_V(x,y,z,0)=f_{cV}(x,y,z). \quad (7)$$

Here $D_{cI}(x,y,z,T)$, $D_{cV}(x,y,z,T)$, $D_{cIS}(x,y,z,T)$ and $D_{cVS}(x,y,z,T)$ are the coefficients of volumetric and surficial diffusions of complexes of radiation defects; $k_I(x,y,z,T)$ and $k_V(x,y,z,T)$ are the parameters of decay of complexes of radiation defects.

Chemical potential « in Eq.(1) could be determine by the following relation [8]

$$« =E(z)\Omega\pm_{ij}\,[u_{ij}(x,y,z,t)+u_{ji}(x,y,z,t)]/2.$$

Here $E(z)$ is the Young modulus, $\pm_{ij}$ is the stress tensor; $u_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)$ is the deformation tensor; u_i, u_j are the components $u_x(x,y,z,t)$, $u_y(x,y,z,t)$ and $u_z(x,y,z,t)$ of the displacement vector $\vec{u}(x,y,z,t)$; x_i, x_j are the coordinate x, y, z. The previous equation could be transform to the following form

$$« (x,y,z,t)=\left[\frac{\partial u_i(x,y,z,t)}{\partial x_j}+\frac{\partial u_j(x,y,z,t)}{\partial x_i}\right]\left\{\frac{1}{2}\left[\frac{\partial u_i(x,y,z,t)}{\partial x_j}+\frac{\partial u_j(x,y,z,t)}{\partial x_i}\right]-\right.$$

$$\left.-£_0¢_{ij}+\frac{\pm(z)¢_{ij}}{1-2\pm(z)}\left[\frac{\partial u_k(x,y,z,t)}{\partial x_k}-3£_0\right]-K(z)\quad(z)[T(x,y,z,t)-T_0]\,¢_{ij}\right\}\frac{\Omega}{2}E(z)$$

.

Here ± is Poisson coefficient; $£_0=(a_s-a_{EL})/a_{EL}$ is the mismatch parameter; a_s, a_{EL} are lattice distances of the substrate and the epitaxial layer; K is the modulus of uniform compression; is the coefficient of thermal expansion; T_r is the equilibrium temperature, which coincide (for our case) with room temperature. Components of displacement vector could be obtained by solution of the following equations [9]

$$\begin{cases}\circ\,(z)\frac{\partial^2 u_x(x,y,z,t)}{\partial t^2}=\frac{\partial\pm_{xx}(x,y,z,t)}{\partial x}+\frac{\partial\pm_{xy}(x,y,z,t)}{\partial y}+\frac{\partial\pm_{xz}(x,y,z,t)}{\partial z}\\ \circ\,(z)\frac{\partial^2 u_y(x,y,z,t)}{\partial t^2}=\frac{\partial\pm_{yx}(x,y,z,t)}{\partial x}+\frac{\partial\pm_{yy}(x,y,z,t)}{\partial y}+\frac{\partial\pm_{yz}(x,y,z,t)}{\partial z}\\ \circ\,(z)\frac{\partial^2 u_z(x,y,z,t)}{\partial t^2}=\frac{\partial\pm_{zx}(x,y,z,t)}{\partial x}+\frac{\partial\pm_{zy}(x,y,z,t)}{\partial y}+\frac{\partial\pm_{zz}(x,y,z,t)}{\partial z}\end{cases}\qquad(3)$$

Here

$$\pm_{ij}=\frac{E(z)}{2[1+\pm(z)]}\left[\frac{\partial u_i(x,y,z,t)}{\partial x_j}+\frac{\partial u_j(x,y,z,t)}{\partial x_i}-\frac{¢_{ij}}{3}\frac{\partial u_k(x,y,z,t)}{\partial x_k}\right]+K(z)¢_{ij}\times$$

$$\times\frac{\partial u_k(x,y,z,t)}{\partial x_k}-\quad(z)K(z)[T(x,y,z,t)-T_r],$$ $\circ\,(z)$ is the density of materials of heterostructure, $¢_{ij}$ Is the Kronecker symbol. With account the relation for $\pm_{ij}$ last system of equation could be written as

$$\circ\,(z)\frac{\partial^2 u_x(x,y,z,t)}{\partial t^2}=\left\{K(z)+\frac{5E(z)}{6[1+\pm(z)]}\right\}\frac{\partial^2 u_x(x,y,z,t)}{\partial x^2}+\left\{K(z)-\frac{E(z)}{3[1+\pm(z)]}\right\}\times$$

$$\times\frac{\partial^2 u_y(x,y,z,t)}{\partial x\partial y}+\frac{E(z)}{2[1+\pm(z)]}\left[\frac{\partial^2 u_y(x,y,z,t)}{\partial y^2}+\frac{\partial^2 u_z(x,y,z,t)}{\partial z^2}\right]+\left[K(z)+\frac{E(z)}{3[1+\pm(z)]}\right]\times$$

$$\times\frac{\partial^2 u_z(x,y,z,t)}{\partial x\partial z}-K(z)\quad(z)\frac{\partial T(x,y,z,t)}{\partial x}$$

$$\circ(z)\frac{\partial^2 u_y(x,y,z,t)}{\partial t^2}=\frac{E(z)}{2[1+\pm(z)]}\left[\frac{\partial^2 u_y(x,y,z,t)}{\partial x^2}+\frac{\partial^2 u_x(x,y,z,t)}{\partial x\partial y}\right]-\frac{\partial T(x,y,z,t)}{\partial y}\times$$

$$\times K(z)\quad(z)+\frac{\partial}{\partial z}\left\{\frac{E(z)}{2[1+\pm(z)]}\left[\frac{\partial u_y(x,y,z,t)}{\partial z}+\frac{\partial u_z(x,y,z,t)}{\partial y}\right]\right\}+\frac{\partial^2 u_y(x,y,z,t)}{\partial y^2}\times$$

$$(3a)$$

$$\times\left\{\frac{5E(z)}{12[1+\pm(z)]}+K(z)\right\}+\left\{K(z)-\frac{E(z)}{6[1+\pm(z)]}\right\}\frac{\partial^2 u_y(x,y,z,t)}{\partial y\partial z}+K(z)\frac{\partial^2 u_y(x,y,z,t)}{\partial x\partial y}$$

$$\circ(z)\frac{\partial^2 u_z(x,y,z,t)}{\partial t^2}=\frac{E(z)}{2[1+\pm(z)]}\left[\frac{\partial^2 u_z(x,y,z,t)}{\partial x^2}+\frac{\partial^2 u_z(x,y,z,t)}{\partial y^2}+\frac{\partial^2 u_x(x,y,z,t)}{\partial x\partial z}+\right.$$

$$\left.+\frac{\partial^2 u_y(x,y,z,t)}{\partial y\partial z}\right]+\frac{\partial}{\partial z}\left\{K(z)\left[\frac{\partial u_x(x,y,z,t)}{\partial x}+\frac{\partial u_y(x,y,z,t)}{\partial y}+\frac{\partial u_x(x,y,z,t)}{\partial z}\right]\right\}+$$

$$+\frac{1}{6}\frac{\partial}{\partial z}\left\{\frac{E(z)}{1+\pm(z)}\left[6\frac{\partial u_z(x,y,z,t)}{\partial z}-\frac{\partial u_x(x,y,z,t)}{\partial x}-\frac{\partial u_y(x,y,z,t)}{\partial y}-\frac{\partial u_z(x,y,z,t)}{\partial z}\right]\right\}-$$

$$-K(z)\quad(z)\frac{\partial T(x,y,z,t)}{\partial z}.$$

Conditions for the system of Eq. ($3a$) could be written in the form

$$\left.\frac{\partial\,\vec{u}(x,y,z,t)}{\partial x}\right|_{x=0}=0;\ \left.\frac{\partial\,\vec{u}(x,y,z,t)}{\partial x}\right|_{x=L_x}=0;\ \left.\frac{\partial\,\vec{u}(x,y,z,t)}{\partial y}\right|_{y=0}=0;$$

$$\left.\frac{\partial\,\vec{u}(x,y,z,t)}{\partial y}\right|_{y=L_y}=0;$$

$$\left.\frac{\partial\,\vec{u}(x,y,z,t)}{\partial z}\right|_{z=0}=0;\ \left.\frac{\partial\,\vec{u}(x,y,z,t)}{\partial z}\right|_{z=L_z}=0;\ \vec{u}(x,y,z,0)=\vec{u}_0;\ \vec{u}(x,y,z,\infty)=\vec{u}_0.$$

In relation ($3a$), both the nonstationarity of the temperature field and its inhomogeneity are taken into account. The space-time temperature distribution can be described using the corresponding Fourier law [10]

$$c\,(T)\frac{\partial T(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[{}^{a}(x,y,z,T)\frac{\partial T(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[{}^{a}(x,y,z,T)\frac{\partial T(x,y,z,t)}{\partial y}\right]+$$

$$+\frac{\partial}{\partial z}\left[{}^{a}(x,y,z,T)\frac{\partial T(x,y,z,t)}{\partial z}\right]+p(x,y,z,t) \tag{4}$$

with initial and boundary conditions

$$\left.\frac{\partial T(x,y,z,t)}{\partial x}\right|_{x=0}=0;\ \left.\frac{\partial T(x,y,z,t)}{\partial x}\right|_{x=L_x}=0;\ \left.\frac{\partial T(x,y,z,t)}{\partial y}\right|_{y=0}=0;$$

$$\left.\frac{\partial T(x,y,z,t)}{\partial y}\right|_{y=L_y}=0;$$

$$\left.\frac{\partial T(x,y,z,t)}{\partial z}\right|_{z=0}=0;\ \left.\frac{\partial T(x,y,z,t)}{\partial z}\right|_{z=L_z}=0;\ T(x,y,z,0)=T_r.$$

Here ^{a} is the thermal conductivity coefficient. Value of the coefficient depend on properties of the heterostructure materials and on value of temperature. The dependence of the thermal conductivity coefficient on temperature in the considered interval of temperature could be approximated as follows: ${}^{a}(x,y,z,T)={}^{a}_{ass}(x,y,z)[1+« T_d^{\,¨}/T^{\,¨}(x,y,z,t)]$ (see, for example, [10]). $c(T)=c_{ass}[1$-g $exp(-T(x,y,z,t)/T_d)]$ is the heat capacity of the material; T_d is the Debye temperature [10]. In the most interesting interval of values the considered temperature is comparable with the Debye temperature or exceeds it. In this situation one can consider: $c(T)\approx c_{ass}$. $p(x,y,z,t)$ is the bulk power density, which released in the heterostructure during annealing. At the first stage let us calculate spatio-temporal distribution of temperature. Let us solve equation (4) by method of averaging of functional corrections [11-14]. Framework this method we replace the spatio-temporal distribution of temperature on the right-hand side of the equation (4) by its not yet known avearge value $\sim_{1T}$. The replacement gives a possibility to obtain the equation for calculation of the first-order approximation of the considered temperature in the following form

$$c_{ass}\frac{\partial T_1(x,y,z,t)}{\partial t}=p(x,y,z,t). \tag{5}$$

Integration of left and right sides of Eq. (5) on time t leads to the following result

$$T_1(x,y,z,t)=\frac{1}{c_{ass}}\int\limits_0^t p\,(x,y,z,^2)d^2+T_r. \tag{6}$$

Not yet known average value $\sim_{1T}$ could be determine by standard relation [11-13]

$$\sim_{1T}=\frac{1}{\Theta L_xL_yL_z}\int\limits_0^{\Theta}\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z}T_1(x,y,z,t)\,dz\,dy\,dx\,dt. \tag{7}$$

The second-order approximation of temperature could be obtained by the following replacement $T(x,y,z,t)\rightarrow\sim_{T2}+T_1(x,y,z,t)$ in the right side of Eq. (4). Appropriate equation could be written as

$$c_{ass}\frac{\partial T_2(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left(\text{a}_{ass}(x,y,z)\left\{1+\frac{\text{«}\,T_d^{\text{¨}}}{[\sim_{2T}+T_1(x,y,z,t)]^{\text{¨}}}\right\}\frac{\partial T_1(x,y,z,t)}{\partial x}\right)+$$

$$+\frac{\partial}{\partial y}\left(\text{a}_{ass}(x,y,z)\left\{1+\frac{\text{«}\,T_d^{\text{¨}}}{[\sim_{2T}+T_1(x,y,z,t)]^{\text{¨}}}\right\}\frac{\partial T_1(x,y,z,t)}{\partial y}\right)+$$

$$+\frac{\partial}{\partial z}\left(\text{a}_{ass}(x,y,z)\left\{1+\frac{\text{«}\,T_d^{\text{¨}}}{[\sim_{2T}+T_1(x,y,z,t)]^{\text{¨}}}\right\}\frac{\partial T_1(x,y,z,t)}{\partial z}\right)+p(x,y,z,t) \tag{8}$$

By differentiating term by term, we transform the above equation to the following form

$$c_{ass}\frac{\partial T_2(x,y,z,t)}{\partial t}=\left\{1+\frac{\text{«}\,T_d^{\text{¨}}}{[\sim_{2T}+T_1(x,y,z,t)]^{\text{¨}}}\right\}\frac{\partial T_1(x,y,z,t)}{\partial x}\frac{\partial\,\text{a}_{ass}(x,y,z)}{\partial x}-\text{a}_{ass}(x,y,z)\times$$

$$\times\left[\frac{\partial T_1(x,y,z,t)}{\partial x}\right]^2\frac{\text{¨}\,\text{«}\,T_d^{\text{¨}}}{[\sim_{2T}+T_1(x,y,z,t)]^{\text{¨}+1}}+\left\{1+\frac{\text{«}\,T_d^{\text{¨}}}{[\sim_{2T}+T_1(x,y,z,t)]^{\text{¨}}}\right\}\frac{\partial^2 T_1(x,y,z,t)}{\partial x^2}\times$$

$$\times\,\text{a}_{ass}(x,y,z)+\left\{1+\frac{\text{«}\,T_d^{\text{¨}}}{[\sim_{2T}+T_1(x,y,z,t)]^{\text{¨}}}\right\}\frac{\partial T_1(x,y,z,t)}{\partial y}\frac{\partial\,\text{a}_{ass}(x,y,z)}{\partial y}-\left[\frac{\partial T_1(x,y,z,t)}{\partial y}\right]^2\times$$

$$\times\frac{\text{¨}\,\text{«}\,T_d^{\text{¨}}\,\text{a}_{ass}(x,y,z)}{[\sim_{2T}+T_1(x,y,z,t)]^{\text{¨}+1}}+\text{a}_{ass}(x,y,z)\left\{1+\frac{\text{«}\,T_d^{\text{¨}}}{[\sim_{2T}+T_1(x,y,z,t)]^{\text{¨}}}\right\}\frac{\partial^2 T_1(x,y,z,t)}{\partial y^2}+$$

$$+\frac{\partial\,\text{a}_{ass}(x,y,z)}{\partial z}\frac{\partial T_1(x,y,z,t)}{\partial z}\left\{1+\frac{\text{«}\,T_d^{\text{¨}}}{[\sim_{2T}+T_1(x,y,z,t)]^{\text{¨}}}\right\}-\frac{\text{¨}\,\text{«}\,T_d^{\text{¨}}\,\text{a}_{ass}(x,y,z)}{[\sim_{2T}+T_1(x,y,z,t)]^{\text{¨}+1}}\times$$

$$+\,\text{a}_{ass}(x,y,z)\left\{1+\frac{\text{«}\,T_d^{\text{¨}}}{[\sim_{2T}+T_1(x,y,z,t)]^{\text{¨}}}\right\}\frac{\partial^2 T_1(x,y,z,t)}{\partial z^2}+p(x,y,z,t). \tag{8a}$$

Integration of the above equation on time gives a possibility to obtain equation for calculation of the second-order approximation of temperature in the following form

$$c_{ass}T_2(x,y,z,t)=\int_0^t\left\{1+\frac{\text{«}\,T_d^{\text{¨}}}{[\sim_{2T}+T_1(x,y,z,\text{²})]^{\text{¨}}}\right\}\frac{\partial T_1(x,y,z,\text{²})}{\partial x}d\,\text{²}\,\frac{\partial\,\text{a}_{ass}(x,y,z)}{\partial x}-\text{¨}\,\text{«}\,T_d^{\text{¨}}\times$$

$$\times\int_0^t\left[\frac{\partial T_1(x,y,z,\text{²})}{\partial x}\right]^2\frac{\text{a}_{ass}(x,y,z)\,d\,\text{²}}{[\sim_{2T}+T_1(x,y,z,\text{²})]^{\text{¨}+1}}+\text{a}_{ass}(x,y,z)\int_0^t\left\{1+\frac{\text{«}\,T_d^{\text{¨}}}{[\sim_{2T}+T_1(x,y,z,\text{²})]^{\text{¨}}}\right\}\times$$

$$\times\frac{\partial^2 T_1(x,y,z,\text{²})}{\partial x^2}d\,\text{²}+\frac{\partial\,\text{a}_{ass}(x,y,z)}{\partial y}\int_0^t\left\{1+\frac{\text{«}\,T_d^{\text{¨}}}{[\sim_{2T}+T_1(x,y,z,\text{²})]^{\text{¨}}}\right\}\frac{\partial T_1(x,y,z,\text{²})}{\partial y}d\,\text{²}-$$

$$-\int_0^t\left[\frac{\partial T_1(x,y,z,\text{²})}{\partial y}\right]^2\frac{\text{¨}\,\text{«}\,T_d^{\text{¨}}\,\text{a}_{ass}(x,y,z)}{[\sim_{2T}+T_1(x,y,z,\text{²})]^{\text{¨}+1}}d\,\text{²}+\text{a}_{ass}(x,y,z)\int_0^t\left\{1+\frac{\text{«}\,T_d^{\text{¨}}}{[\sim_{2T}+T_1(x,y,z,\text{²})]^{\text{¨}}}\right\}\times$$

$$\times\frac{\partial^2 T_1(x,y,z,{}^2)}{\partial y^2}d^2+\frac{\partial\, a_{ass}(x,y,z)}{\partial z}\int_0^t\left\{1+\frac{\ll T_d^{\ddot{}}}{\left[\sim_{2T}+T_1(x,y,z,{}^2)\right]^{\ddot{}}}\right\}\frac{\partial\, T_1(x,y,z,{}^2)}{\partial z}d^2-$$

$$-\ddot{}\ll T_d^{\ddot{}}\, a_{ass}(x,y,z)\int_0^t\left[\frac{\partial\, T_1(x,y,z,{}^2)}{\partial z}\right]^2\frac{1}{\left[\sim_{2T}+T_1(x,y,z,{}^2)\right]^{\ddot{}+1}}d^2+\int_0^t p(x,y,z,{}^2)d^2+$$

$$+a_{ass}(x,y,z)\int_0^t\left\{1+\frac{\ll T_d^{\ddot{}}}{\left[\sim_{2T}+T_1(x,y,z,{}^2)\right]^{\ddot{}}}\right\}\frac{\partial^2 T_1(x,y,z,{}^2)}{\partial z^2}d^2\,. \qquad (8b)$$

Average value of the second-order approximation of temperature $\sim_T$ could be determined by the following standard relation [11-13]

$$\sim_{2T}=\frac{1}{\Theta L_x L_y L_z}\int_0^{\Theta}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\left[\,T_2(x,y,z,t)-T_1(x,y,z,t)\right]d\,z\,d\,y\,d\,x\,d\,t\,. \qquad (9)$$

Appropriate equation to calculate the average value $\sim_{T2}$ could be written as

$$\int_0^{\Theta}(\Theta-t)\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\left\{1+\frac{\ll T_d^{\ddot{}}}{\left[\sim_{2T}+T_1(x,y,z,t)\right]^{\ddot{}}}\right\}\frac{\partial\, T_1(x,y,z,t)}{\partial x}d\,z\,d\,y\,d\,x\,d\,t\frac{\partial\, a_{ass}(x,y,z)}{\partial x}-\ll T_d^{\ddot{}}\times$$

$$\times\ddot{}\int_0^{\Theta}(\Theta-t)\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}a_{ass}(x,y,z)\left[\frac{\partial\, T_1(x,y,z,t)}{\partial x}\right]^2\frac{d\,z\,d\,y\,d\,x\,d\,t}{\left[\sim_{2T}+T_1(x,y,z,t)\right]^{\ddot{}+1}}+\int_0^{\Theta}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}a_{ass}(x,y,z)\times$$

$$\times\left\{1+\frac{\ll T_d^{\ddot{}}}{\left[\sim_{2T}+T_1(x,y,z,t)\right]^{\ddot{}}}\right\}\frac{\partial^2 T_1(x,y,z,t)}{\partial x^2}d\,z\,d\,y\,d\,x(\Theta-t)\,d\,t+\int_0^{\Theta}(\Theta-t)\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\frac{\partial\, a_{ass}(x,y,z)}{\partial y}\times$$

$$\times\left\{1+\frac{\ll T_d^{\ddot{}}}{\left[\sim_{2T}+T_1(x,y,z,t)\right]^{\ddot{}}}\right\}\frac{\partial\, T_1(x,y,z,t)}{\partial y}\,d\,z\,d\,y\,d\,x\,d\,t-\ddot{}\ll\int_0^{\Theta}(\Theta-t)\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}T_d^{\ddot{}}\times$$

$$\times\left[\frac{\partial\, T_1(x,y,z,t)}{\partial y}\right]^2\frac{a_{ass}(x,y,z)d\,z\,d\,y\,d\,x}{\left[\sim_{2T}+T_1(x,y,z,{}^2)\right]^{\ddot{}+1}}d\,t+\int_0^{\Theta}(\Theta-t)\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\left\{1+\frac{\ll T_d^{\ddot{}}}{\left[\sim_{2T}+T_1(x,y,z,t)\right]^{\ddot{}}}\right\}\times$$

$$\times\, a_{ass}(x,y,z)\frac{\partial^2 T_1(x,y,z,t)}{\partial y^2}d\,z\,d\,y\,d\,x\,d\,t+\int_0^{\Theta}(\Theta-t)\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\left\{1+\frac{\ll T_d^{\ddot{}}}{\left[\sim_{2T}+T_1(x,y,z,t)\right]^{\ddot{}}}\right\}\times$$

$$\times\frac{\partial\, T_1(x,y,z,t)}{\partial z}\frac{\partial\, a_{ass}(x,y,z)}{\partial z}d\,z\,d\,y\,d\,x\,d\,t-\ddot{}\ll T_d^{\ddot{}}\int_0^{\Theta}(\Theta-t)\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\left[\frac{\partial\, T_1(x,y,z,t)}{\partial z}\right]^2\times$$

$$\times\frac{a_{ass}(x,y,z)}{\left[\sim_{2T}+T_1(x,y,z,t)\right]^{\ddot{}+1}}\,d\,z\,d\,y\,d\,x\,d\,t+\int_0^{\Theta}(\Theta-t)\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}p(x,y,z,t)\,d\,z\,d\,y\,d\,x\,d\,t+ \qquad (10)$$

$$+\int_0^{\Theta}(\Theta-t)\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}a_{ass}(x,y,z)\left\{1+\frac{\ll T_d^{\ddot{}}}{\left[\sim_{2T}+T_1(x,y,z,t)\right]^{\ddot{}}}\right\}\frac{\partial^2 T_1(x,y,z,t)}{\partial z^2}d\,z\,d\,y\,d\,x\,d\,t=0.$$

Solution of Eq. (10) (i.e. average value $\sim_{T\ 2}$) depends on value of parameter ¨. Next let us calculate solutions of equations of system (3*a*). These solutions describe components of displacement vector. To determine the first-order approximations of these components by using method of averaging functional corrections, we substitute the not yet known average values $\sim_i$ into the right side of the equations under consideration. This substitution leads to the following result

$$\circ\,(z)\frac{\partial^2 u_{1x}(x,y,z,t)}{\partial t^2}=-K(z)\quad(z)\frac{\partial T(x,y,z,t)}{\partial x},$$

$$\circ\,(z)\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial t^2}=-K(z)\quad(z)\frac{\partial T(x,y,z,t)}{\partial y},$$

$$\circ\,(z)\frac{\partial^2 u_{1z}(x,y,z,t)}{\partial t^2}=-K(z)\quad(z)\frac{\partial T(x,y,z,t)}{\partial z}.$$

Integration of left and right sides of the above equations on time *t* leads to the following result

$$u_{1x}(x,y,z,t)=K(z)\frac{(z)}{\circ\,(z)}\frac{\partial}{\partial x}\int\limits_0^t\int\limits_0^{\text{g}}T(x,y,z,{}^2)d{}^2\,d\text{g}-$$

$$-K(z)\frac{(z)}{\circ\,(z)}\frac{\partial}{\partial x}\int\limits_0^{\infty}\int\limits_0^{\text{g}}T(x,y,z,{}^2)d{}^2\,d\text{g}+u_{0x},$$

$$u_{1y}(x,y,z,t)=K(z)\frac{(z)}{\circ\,(z)}\frac{\partial}{\partial y}\int\limits_0^t\int\limits_0^{\text{g}}T(x,y,z,{}^2)d{}^2\,d\text{g}-$$

$$-K(z)\frac{(z)}{\circ\,(z)}\frac{\partial}{\partial y}\int\limits_0^{\infty}\int\limits_0^{\text{g}}T(x,y,z,{}^2)d{}^2\,d\text{g}+u_{0y},$$

$$u_{1z}(x,y,z,t)=K(z)\frac{(z)}{\circ\,(z)}\frac{\partial}{\partial z}\int\limits_0^t\int\limits_0^{\text{g}}T(x,y,z,{}^2)d{}^2\,d\text{g}-$$

$$-K(z)\frac{(z)}{\circ\,(z)}\frac{\partial}{\partial z}\int\limits_0^{\infty}\int\limits_0^{\text{g}}T(x,y,z,{}^2)d{}^2\,d\text{g}+u_{0z}.$$

Approximations with the second and higher orders of components of displacement vector could be obtained by standard replacement of required functions in right sides of Eqs. (8) on the following sum $\sim_i+u_i(x,y,z,t)$ [11-13]. The replacement leads to the following result

$$\circ\,(z)\frac{\partial^2 u_{2x}(x,y,z,t)}{\partial t^2}=\left\{K(z)+\frac{5E(z)}{6[1+\pm\,(z)]}\right\}\frac{\partial^2 u_{1x}(x,y,z,t)}{\partial x^2}+\left\{K(z)-\frac{E(z)}{3[1+\pm\,(z)]}\right\}\times$$

$$\times\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial x\partial y}+\frac{E(z)}{2[1+\pm\,(z)]}\left[\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial y^2}+\frac{\partial^2 u_{1z}(x,y,z,t)}{\partial z^2}\right]+\left\{K(z)+\frac{E(z)}{3[1+\pm\,(z)]}\right\}\times$$

$$\times\frac{\partial^2 u_{1z}(x,y,z,t)}{\partial x\partial z}-K(z)\quad(z)\frac{\partial T(x,y,z,t)}{\partial x}$$

$$\circ\,(z)\frac{\partial^2 u_{2y}(x,y,z,t)}{\partial t^2}=\frac{E(z)}{2[1+\pm\,(z)]}\left[\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial x^2}+\frac{\partial^2 u_{1x}(x,y,z,t)}{\partial x\partial y}\right]-K(z)\quad(z)\times$$

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$$\times\frac{\partial T\left(x,y,z,t\right)}{\partial y}+\frac{\partial}{\partial z}\left\{\frac{E\left(z\right)}{2\left[1+\pm\left(z\right)\right]}\left[\frac{\partial u_{1y}\left(x,y,z,t\right)}{\partial z}+\frac{\partial u_{1z}\left(x,y,z,t\right)}{\partial y}\right]\right\}+\frac{\partial^2 u_{1y}\left(x,y,z,t\right)}{\partial y^2}\times$$

$$\times\left\{\frac{5E\left(z\right)}{12\left[1+\pm\left(z\right)\right]}+K\left(z\right)\right\}+\left\{K\left(z\right)-\frac{E\left(z\right)}{6\left[1+\pm\left(z\right)\right]}\right\}\frac{\partial^2 u_{1y}\left(x,y,z,t\right)}{\partial y\partial z}+K\left(z\right)\frac{\partial^2 u_{1y}\left(x,y,z,t\right)}{\partial x\partial y}$$

$$\circ\left(z\right)\frac{\partial^2 u_{2z}\left(x,y,z,t\right)}{\partial t^2}=\frac{E\left(z\right)}{2\left[1+\pm\left(z\right)\right]}\left[\frac{\partial^2 u_{1z}\left(x,y,z,t\right)}{\partial x^2}+\frac{\partial^2 u_{1z}\left(x,y,z,t\right)}{\partial y^2}+\frac{\partial^2 u_{1x}\left(x,y,z,t\right)}{\partial x\partial z}+\right.$$

$$\left.+\frac{\partial^2 u_{1y}\left(x,y,z,t\right)}{\partial y\partial z}\right]+\frac{\partial}{\partial z}\left\{K\left(z\right)\left[\frac{\partial u_{1x}\left(x,y,z,t\right)}{\partial x}+\frac{\partial u_{1y}\left(x,y,z,t\right)}{\partial y}+\frac{\partial u_{1x}\left(x,y,z,t\right)}{\partial z}\right]\right\}+$$

$$+\frac{1}{6}\frac{\partial}{\partial z}\left\{\frac{E\left(z\right)}{1+\pm\left(z\right)}\left[6\frac{\partial u_{1z}\left(x,y,z,t\right)}{\partial z}-\frac{\partial u_{1x}\left(x,y,z,t\right)}{\partial x}-\frac{\partial u_{1y}\left(x,y,z,t\right)}{\partial y}-\frac{\partial u_{1z}\left(x,y,z,t\right)}{\partial z}\right]\right\}-$$

$$-K\left(z\right)\quad\left(z\right)\frac{\partial T\left(x,y,z,t\right)}{\partial z}.$$

Integration of left and right sides of the above equations on time *t* leads to the following result

$$u_{2x}\left(x,y,z,t\right)=\frac{1}{\circ\left(z\right)}\left\{K\left(z\right)+\frac{5E\left(z\right)}{6\left[1+\pm\left(z\right)\right]}\right\}\frac{\partial^2}{\partial x^2}\int\limits_0^t\int\limits_0^{g}u_{1x}\left(x,y,z,^2\right)d^2\,d\mathrm{g}+\frac{1}{\circ\left(z\right)}\left\{K\left(z\right)-\right.$$

$$\left.-\frac{E\left(z\right)}{3\left[1+\pm\left(z\right)\right]}\right\}\frac{\partial^2}{\partial x\partial y}\int\limits_0^t\int\limits_0^{g}u_{1y}\left(x,y,z,^2\right)d^2\,d\mathrm{g}+\frac{E\left(z\right)}{2\circ\left(z\right)}\left[\frac{\partial^2}{\partial y^2}\int\limits_0^t\int\limits_0^{g}u_{1y}\left(x,y,z,^2\right)d^2\,d\mathrm{g}+\right.$$

$$\left.+\frac{\partial^2}{\partial z^2}\int\limits_0^t\int\limits_0^{g}u_{1z}\left(x,y,z,^2\right)d^2\,d\mathrm{g}\right]\frac{1}{1+\pm\left(z\right)}+\frac{1}{\circ\left(z\right)}\frac{\partial^2}{\partial x\partial z}\int\limits_0^t\int\limits_0^{g}u_{1z}\left(x,y,z,^2\right)d^2\,d\mathrm{g}\left\{K\left(z\right)+\right.$$

$$\left.+\frac{E\left(z\right)}{3\left[1+\pm\left(z\right)\right]}\right\}-K\left(z\right)\frac{\left(z\right)}{\circ\left(z\right)}\frac{\partial}{\partial x}\int\limits_0^t\int\limits_0^{g}T\left(x,y,z,^2\right)d^2\,d\mathrm{g}-\frac{1}{\circ\left(z\right)}\left\{K\left(z\right)+\frac{5E\left(z\right)}{6\left[1+\pm\left(z\right)\right]}\right\}\times$$

$$\times\frac{\partial^2}{\partial x^2}\int\limits_0^\infty\int\limits_0^{g}u_{1x}\left(x,y,z,^2\right)d^2\,d\mathrm{g}-\frac{1}{\circ\left(z\right)}\left\{K\left(z\right)-\frac{E\left(z\right)}{3\left[1+\pm\left(z\right)\right]}\right\}\frac{\partial^2}{\partial x\partial y}\int\limits_0^\infty\int\limits_0^{g}u_{1y}\left(x,y,z,^2\right)d^2\,d\mathrm{g}-$$

$$-\frac{E\left(z\right)}{2\circ\left(z\right)\left[1+\pm\left(z\right)\right]}\left[\frac{\partial^2}{\partial y^2}\int\limits_0^\infty\int\limits_0^{g}u_{1y}\left(x,y,z,^2\right)d^2\,d\mathrm{g}+\frac{\partial^2}{\partial z^2}\int\limits_0^\infty\int\limits_0^{g}u_{1z}\left(x,y,z,^2\right)d^2\,d\mathrm{g}\right]-\frac{1}{\circ\left(z\right)}\times$$

$$\times\left\{K\left(z\right)+\frac{E\left(z\right)}{3\left[1+\pm\left(z\right)\right]}\right\}\frac{\partial^2}{\partial x\partial z}\int\limits_0^\infty\int\limits_0^{g}u_{1z}\left(x,y,z,^2\right)d^2\,d\mathrm{g}+u_{0x}+\frac{\partial}{\partial x}\int\limits_0^\infty\int\limits_0^{g}T\left(x,y,z,^2\right)d^2\,d\mathrm{g}\times$$

$$\times K(z)\quad (z)/{}^\circ(z)$$

$$u_{2y}(x,y,z,t)=\frac{E(z)}{2\,{}^\circ(z)}\Bigg[\frac{\partial^2}{\partial x^2}\int_0^t\int_0^g u_{1x}(x,y,z,{}^2)\,d^2\,dg+\frac{\partial^2}{\partial x\partial y}\int_0^t\int_0^g u_{1x}(x,y,z,{}^2)\,d^2\,dg\Bigg]\times$$

$$\times\frac{1}{1+\pm(z)}+\frac{K(z)}{{}^\circ(z)}\frac{\partial^2}{\partial x\partial y}\int_0^t\int_0^g u_{1y}(x,y,z,{}^2)\,d^2\,dg+\frac{1}{{}^\circ(z)}\frac{\partial^2}{\partial y^2}\int_0^t\int_0^g u_{1x}(x,y,z,{}^2)\,d^2\,dg\times$$

$$\times\Bigg\{\frac{5E(z)}{12[1+\pm(z)]}+K(z)\Bigg\}+\frac{\partial}{\partial z}\Bigg\{\Bigg[\frac{\partial}{\partial z}\int_0^t\int_0^g u_{1y}(x,y,z,{}^2)\,d^2\,dg+\frac{\partial}{\partial y}\int_0^t\int_0^g u_{1z}(x,y,z,{}^2)\,d^2\,dg\Bigg]\times$$

$$\times\frac{E(z)}{1+\pm(z)}\Bigg\}\frac{1}{2\,{}^\circ(z)}-K(z)\frac{(z)}{{}^\circ(z)}\int_0^t\int_0^g T\,(x,y,z,{}^2)\,d^2\,dg+\frac{1}{{}^\circ(z)}\Bigg\{K(z)-\frac{E(z)}{6[1+\pm(z)]}\Bigg\}\times$$

$$\times\frac{\partial^2}{\partial y\partial z}\int_0^t\int_0^g u_{1y}(x,y,z,{}^2)\,d^2\,dg-\frac{E(z)}{2\,{}^\circ(z)[1+\pm(z)]}\Bigg[\frac{\partial^2}{\partial x^2}\int_0^\infty\int_0^g u_{1x}(x,y,z,{}^2)\,d^2\,dg+$$

$$+\frac{\partial^2}{\partial x\partial y}\int_0^\infty\int_0^g u_{1x}(x,y,z,{}^2)\,d^2\,dg\Bigg]-K(z)\frac{(z)}{{}^\circ(z)}\int_0^\infty\int_0^g T\,(x,y,z,{}^2)\,d^2\,dg-\frac{K(z)}{{}^\circ(z)}\times$$

$$\times\frac{\partial^2}{\partial x\partial y}\int_0^\infty\int_0^g u_{1y}(x,y,z,{}^2)\,d^2\,dg-\Bigg\{K(z)+\frac{5E(z)}{12[1+\pm(z)]}\Bigg\}\frac{\partial^2}{\partial y^2}\int_0^\infty\int_0^g u_{1x}(x,y,z,{}^2)\,d^2\,dg\times$$

$$\times\frac{1}{{}^\circ(z)}-\frac{1}{2\,{}^\circ(z)}\frac{\partial}{\partial z}\Bigg\{\frac{E(z)}{1+\pm(z)}\Bigg[\frac{\partial}{\partial z}\int_0^\infty\int_0^g u_{1y}(x,y,z,{}^2)\,d^2\,dg+\frac{\partial}{\partial y}\int_0^\infty\int_0^g u_{1z}(x,y,z,{}^2)\,d^2\,dg\Bigg]\Bigg\}-$$

$$-\frac{1}{{}^\circ(z)}\Bigg\{K(z)-\frac{E(z)}{6[1+\pm(z)]}\Bigg\}\frac{\partial^2}{\partial y\partial z}\int_0^\infty\int_0^g u_{1y}(x,y,z,{}^2)\,d^2\,dg+u_{0y}$$

$$u_{2z}(x,y,z,t)=\frac{E(z)}{2\,[1+\pm(z)]}\Bigg[\frac{\partial^2}{\partial x^2}\int_0^\infty\int_0^g u_{1z}(x,y,z,{}^2)\,d^2\,dg+\frac{\partial^2}{\partial y^2}\int_0^\infty\int_0^g u_{1z}(x,y,z,{}^2)\,d^2\,dg+$$

$$+\frac{\partial^2}{\partial x\partial z}\int_0^\infty\int_0^g u_{1x}(x,y,z,{}^2)\,d^2\,dg+\frac{\partial^2}{\partial y\partial z}\int_0^\infty\int_0^g u_{1y}(x,y,z,{}^2)\,d^2\,dg\Bigg]\frac{1}{{}^\circ(z)}+\frac{\partial}{\partial z}\Bigg\{K(z)\times$$

$$\times\Bigg[\frac{\partial}{\partial x}\int_0^\infty\int_0^g u_{1x}(x,y,z,{}^2)\,d^2\,dg+\frac{\partial}{\partial y}\int_0^\infty\int_0^g u_{1x}(x,y,z,{}^2)\,d^2\,dg+\frac{\partial}{\partial z}\int_0^\infty\int_0^g u_{1x}(x,y,z,{}^2)\,d^2\,dg\Bigg]\Bigg\}\times$$

$$\times\frac{1}{{}^\circ(z)}+\frac{1}{6\,{}^\circ(z)}\frac{\partial}{\partial z}\Bigg\{\frac{E(z)}{1+\pm(z)}\Bigg[6\frac{\partial}{\partial z}\int_0^\infty\int_0^g u_{1z}(x,y,z,{}^2)\,d^2\,dg-\frac{\partial}{\partial x}\int_0^\infty\int_0^g u_{1x}(x,y,z,{}^2)\,d^2\,dg-$$

$$-\frac{\partial}{\partial y}\int_0^{\infty}\int_0^{g} u_{1y}(x,y,z,{}^2)\,d^2\,d\text{g}--\frac{\partial}{\partial z}\int_0^{\infty}\int_0^{g} u_{1z}(x,y,z,{}^2)\,d^2\,d\text{g}\Bigg]\Bigg]\Bigg\}+u_{0z}-K(z)\frac{(z)}{\circ\,(z)}\times$$

$$\times\frac{\partial}{\partial z}\int_0^{\infty}\int_0^{g} T(x,y,z,{}^2)\,d^2\,d\text{g}\,.$$

We calculate spatio-temporal distributions of concentrations of radiation defects by standard method of averaging of function corrections [11-13]. To use the approach we transform Eqs. (1) and (2) with account initial distributions of the considered concentrations, i.e.

$$\frac{\hat{o}\,I(x,y,z,t)}{\hat{o}\,t}=\frac{\hat{o}}{\hat{o}\,x}\left[D_I(x,y,z,T)\frac{\hat{o}\,I(x,y,z,t)}{\hat{o}\,x}\right]+\frac{\hat{o}}{\hat{o}\,y}\left[D_I(x,y,z,T)\frac{\hat{o}\,I(x,y,z,t)}{\hat{o}\,y}\right]+$$

$$+\frac{\hat{o}}{\hat{o}\,z}\left[D_I(x,y,z,T)\frac{\hat{o}\,I(x,y,z,t)}{\hat{o}\,z}\right]-k_{I,I}(x,y,z,T)I^2(x,y,z,t)+f_I(x,y,z)\text{¢}(t)+$$

$$+\Omega\frac{\partial}{\partial x}\left[\frac{D_{IS}}{kT}\nabla_S\text{«}(x,y,z,t)\int_0^{L_z}I(x,y,W,t)dW\right]-k_{I,V}(x,y,z,T)I(x,y,z,t)V(x,y,z,t)+$$

$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{IS}}{kT}\nabla_S\text{«}(x,y,z,t)\int_0^{L_z}I(x,y,W,t)dW\right]\tag{1a}$$

$$\frac{\hat{o}\,V(x,y,z,t)}{\hat{o}\,t}=\frac{\hat{o}}{\hat{o}\,x}\left[D_V(x,y,z,T)\frac{\hat{o}\,V(x,y,z,t)}{\hat{o}\,x}\right]+\frac{\hat{o}}{\hat{o}\,y}\left[D_V(x,y,z,T)\frac{\hat{o}\,V(x,y,z,t)}{\hat{o}\,y}\right]+$$

$$+\frac{\hat{o}}{\hat{o}\,z}\left[D_V(x,y,z,T)\frac{\hat{o}\,V(x,y,z,t)}{\hat{o}\,z}\right]-k_{V,V}(x,y,z,T)V^2(x,y,z,t)+f_V(x,y,z)\text{¢}(t)+$$

$$+\Omega\frac{\partial}{\partial x}\left[\frac{D_{VS}}{kT}\nabla_S\text{«}(x,y,z,t)\int_0^{L_z}V(x,y,W,t)dW\right]-k_{I,V}(x,y,z,T)I(x,y,z,t)V(x,y,z,t)+$$

$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{VS}}{kT}\nabla_S\text{«}(x,y,z,t)\int_0^{L_z}V(x,y,W,t)dW\right]$$

$$\frac{\hat{o}\,\Phi_I(x,y,z,t)}{\hat{o}\,t}=\frac{\hat{o}}{\hat{o}\,x}\left[D_{\Phi I}(x,y,z,T)\frac{\hat{o}\,\Phi_I(x,y,z,t)}{\hat{o}\,x}\right]+\frac{\hat{o}}{\hat{o}\,y}\left[D_{\Phi I}(x,y,z,T)\frac{\hat{o}\,\Phi_I(x,y,z,t)}{\hat{o}\,y}\right]+$$

$$+\frac{\hat{o}}{\hat{o}\,z}\left[D_{\Phi_I}(x,y,z,T)\frac{\hat{o}\,\Phi_I(x,y,z,t)}{\hat{o}\,z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\text{«}\,(x,y,z,t)\int_0^{L_z}\Phi_I(x,y,W,t)dW\right]+$$

$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\text{«}(x,y,z,t)\int_0^{L_z}\Phi_I(x,y,W,t)dW\right]+k_{I,I}(x,y,z,T)I^2(x,y,z,t)+$$

$$+k_I(x,y,z,T)I(x,y,z,t)+f_{\Phi_I}(x,y,z)\text{¢}(t)\tag{2a}$$

$$\frac{\hat{o}\,\Phi_V(x,y,z,t)}{\hat{o}\,t}=\frac{\hat{o}}{\hat{o}\,x}\left[D_{\Phi_V}(x,y,z,T)\frac{\hat{o}\,\Phi_V(x,y,z,t)}{\hat{o}\,x}\right]+\frac{\hat{o}}{\hat{o}\,y}\left[D_{\Phi_V}(x,y,z,T)\frac{\hat{o}\,\Phi_V(x,y,z,t)}{\hat{o}\,y}\right]+$$

$$+\frac{\hat{o}}{\hat{o}\,z}\left[D_{\Phi_V}(x,y,z,T)\frac{\hat{o}\,\Phi_V(x,y,z,t)}{\hat{o}\,z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\text{«}\,(x,y,z,t)\int_0^{L_z}\Phi_V(x,y,W,t)\,dW\right]+$$

$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\text{«}(x,y,z,t)\int_0^{L_z}\Phi_V(x,y,W,t)\,dW\right]+k_{V,V}(x,y,z,T)V^2(x,y,z,t)+$$

$$+k_V(x,y,z,T)V(x,y,z,t)+f_{\Phi_V}(x,y,z)\text{¢}(t).$$

Further let us replace considered functions in right sides of Eqs. (1a) and (2a) on their not yet known average values $\sim_{1\circ}$. The replacement gives a possibility to obtain equations for the first-order approximations of the considered concentrations

$$\frac{\hat{o}\,I_1(x,y,z,t)}{\hat{o}\,t}=\sim_{1I}z\Omega\frac{\partial}{\partial x}\left[\frac{D_{IS}}{kT}\nabla_S\text{«}\,(x,y,z,t)\right]+\sim_{1I}\Omega\frac{\partial}{\partial y}\left[z\frac{D_{IS}}{kT}\nabla_S\text{«}\,(x,y,z,t)\right]+$$

$$+f_I(x,y,z)\text{¢}(t)-\sim_{1I}^2k_{I,I}(x,y,z,T)-\sim_{1I}\sim_{1V}k_{I,V}(x,y,z,T)\quad(1b)$$

$$\frac{\hat{o}\,V_1(x,y,z,t)}{\hat{o}\,t}=\sim_{1V}z\Omega\frac{\partial}{\partial x}\left[\frac{D_{VS}}{kT}\nabla_S\text{«}(x,y,z,t)\right]+\sim_{1V}\Omega\frac{\partial}{\partial y}\left[z\frac{D_{VS}}{kT}\nabla_S\text{«}\,(x,y,z,t)\right]+$$

$$+f_V(x,y,z)\text{¢}(t)-\sim_{1V}^2k_{V,V}(x,y,z,T)-\sim_{1I}\sim_{1V}k_{I,V}(x,y,z,T)$$

$$\frac{\hat{o}\,\Phi_{1I}(x,y,z,t)}{\hat{o}\,t}=\sim_{1\Phi_I}z\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\text{«}(x,y,z,t)\right]+\sim_{1\Phi_I}z\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\text{«}(x,y,z,t)\right]+$$

$$+f_{\Phi_I}(x,y,z)\text{¢}(t)+k_I(x,y,z,T)I(x,y,z,t)+k_{I,I}(x,y,z,T)I^2(x,y,z,t)\quad(2b)$$

$$\frac{\hat{o}\,\Phi_{1V}(x,y,z,t)}{\hat{o}\,t}=\sim_{1\Phi_V}z\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\text{«}(x,y,z,t)\right]+\sim_{1\Phi_V}z\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\text{«}(x,y,z,t)\right]+$$

$$+f_{\Phi_V}(x,y,z)\text{¢}(t)+k_V(x,y,z,T)V(x,y,z,t)+k_{V,V}(x,y,z,T)\,V^2(x,y,z,t).$$

Integration of left and right sides of the above equations (1b) and (2b) gives a possibility to obtain relations for the first-order approximations of required concentrations in the required form

$$I_1(x,y,z,t)=\sim_{1I}z\Omega\frac{\partial}{\partial x}\int_0^t\frac{D_{IS}}{kT}\nabla_S\text{«}(x,y,z,{}^2)d{}^2+\sim_{1I}z\Omega\frac{\partial}{\partial y}\int_0^t\frac{D_{IS}}{kT}\nabla_S\text{«}(x,y,z,{}^2)d{}^2+$$

$$+f_I(x,y,z)-\sim_{1I}^2\int_0^t k_{I,I}(x,y,z,T)d{}^2-\sim_{1I}\sim_{1V}\int_0^t k_{I,V}(x,y,z,T)d{}^2\quad(1c)$$

$$V_1(x,y,z,t)=\sim_{1V}z\Omega\frac{\partial}{\partial x}\int_0^t\frac{D_{IS}}{kT}\nabla_S\text{«}(x,y,z,{}^2)d{}^2+\sim_{1V}z\Omega\frac{\partial}{\partial y}\int_0^t\frac{D_{IS}}{kT}\nabla_S\text{«}(x,y,z,{}^2)d{}^2+$$

$$+ f_V\left(x,y,z\right) - \sim_{1V}^{2} \int\limits_0^t k_{V,V}\left(x,y,z,T\right) d^{2} - \sim_{1I} \sim_{1V} \int\limits_0^t k_{I,V}\left(x,y,z,T\right) d^{2}$$

$$\Phi_{1I}\left(x,y,z,t\right) = \sim_{1\Phi_I} z\Omega \frac{\partial}{\partial x} \int\limits_0^t \frac{D_{\Phi_I S}}{kT} \nabla_S \text{«}\left(x,y,z,^{2}\right) d^{2} + \Omega \frac{\partial}{\partial x} \int\limits_0^t \frac{D_{\Phi_I S}}{kT} \nabla_S \text{«}\left(x,y,z,^{2}\right) d^{2} \times$$

$$\times z \sim_{1\Phi_I} + f_{\Phi_I}\left(x,y,z\right) + \int\limits_0^t k_I\left(x,y,z,T\right) I\left(x,y,z,^{2}\right) d^{2} + \int\limits_0^t k_{I,I}\left(x,y,z,T\right) I^2\left(x,y,z,^{2}\right) d^{2}$$

$$(2c)$$

$$\Phi_{1V}\left(x,y,z,t\right) = \sim_{1\Phi_V} z\Omega \frac{\partial}{\partial x} \int\limits_0^t \frac{D_{\Phi_V S}}{kT} \nabla_S \text{«}\left(x,y,z,^{2}\right) d^{2} + \Omega \frac{\partial}{\partial x} \int\limits_0^t \frac{D_{\Phi_V S}}{kT} \nabla_S \text{«}\left(x,y,z,^{2}\right) d^{2} \times$$

$$\times \sim_{1\Phi_V} z + f_{\Phi_V}\left(x,y,z\right) + \int\limits_0^t k_V\left(x,y,z,T\right) V\left(x,y,z,^{2}\right) d^{2} + \int\limits_0^t k_{V,V}\left(x,y,z,T\right) V^2\left(x,y,z,^{2}\right) d^{2}$$

We determine average values of the first-order approximations of required concentrations by using standard relations, which is analogous to relation (7) (see, for example, [11-13]). Substitution of relations (1c) and (2c) into relation (7) gives a possibility to obtained required average values in the final form

$$\sim_{1I} = \sqrt{\frac{\left(a_3 + A\right)^2}{4a_4^2} - 4\left(B + \frac{\Theta a_3 B + \Theta^2 L_x L_y L_z a_1}{a_4}\right)} - \frac{a_3 + A}{4a_4},$$

$$\sim_{1V} = \frac{1}{S_{IV00}}\left[\frac{\Theta}{\sim_{1I}} \int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z} f_I\left(x,y,z\right) dz\,dy\,dx - \sim_{1I} S_{II00} - \Theta L_x L_y L_z\right],$$

Where $S_{\circ\circ ij} = \int\limits_0^{\Theta}\left(\Theta - t\right)\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z} k_{\circ,\circ}\left(x,y,z,T\right) I_1^i\left(x,y,z,t\right) V_1^j\left(x,y,z,t\right) dz\,dy\,dx\,dt,$

$a_4 = S_{II00} \times \times \left(S_{IV00}^2 - S_{II00} S_{VV00}\right),$ $a_3 = S_{IV00} S_{II00} + S_{IV00}^2 - S_{II00} S_{VV00},$

$$a_2 = \int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z} f_V\left(x,y,z\right) dz\,dy\,dx \times$$

$$\times S_{IV00} S_{IV00}^2 + 2 S_{VV00} S_{II00} \int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z} f_I\left(x,y,z\right) dz\,dy\,dx - S_{IV00}^2 \int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z} f_I\left(x,y,z\right) dz\,dy\,dx - S_{VV00} \times$$

$$\times \Theta L_x^2 L_y^2 L_z^2 + S_{IV00} \Theta L_x^2 L_y^2 L_z^2,$$

$$a_1 = S_{IV00} \int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z} f_I\left(x,y,z\right) dz\,dy\,dx,\ A = \sqrt{8y + \Theta^2 \frac{a_3^2}{a_4^2} - 4\Theta \frac{a_2}{a_4}},$$

$$a_0 = S_{VV00}\left[\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z} f_I\left(x,y,z\right) dz\,dy\,dx\right]^2,\ B = \frac{\Theta a_2}{6a_4} + \sqrt[3]{\sqrt{q^2 + p^3} - q} - \sqrt[3]{\sqrt{q^2 + p^3} + q},$$

$$q = \frac{\Theta^3 a_2}{24a_4^2}\left(4a_0 - \Theta L_x L_y L_z \frac{a_1 a_3}{a_4}\right) - \frac{a_0 \Theta^2}{8a_4^2}\left(4\Theta a_2 - \Theta^2 \frac{a_3^2}{a_4}\right) - \frac{\Theta^3 a_2^3}{54a_4^3} - L_x^2 L_y^2 L_z^2 \frac{\Theta^4 a_1^2}{8a_4^2},$$

$$p = \Theta^2 \frac{4a_0 a_4 - \Theta L_x L_y L_z a_1 a_3}{12a_4^2} - \frac{\Theta a_2}{18a_4},$$

$$\sim_{1\Phi_I}=\frac{R_{I1}}{\Theta L_xL_yL_z}+\frac{S_{II20}}{\Theta L_xL_yL_z}+\frac{1}{L_xL_yL_z}\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z}f_{\Phi_I}\left(x,y,z\right)d\,z\,d\,y\,d\,x$$

$$\sim_{1\Phi_V}=\frac{R_{V1}}{\Theta L_xL_yL_z}+\frac{S_{VV20}}{\Theta L_xL_yL_z}+\frac{1}{L_xL_yL_z}\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z}f_{\Phi_V}\left(x,y,z\right)d\,z\,d\,y\,d\,x,$$

where $R_{\circ\,i}=\int\limits_0^{\Theta}\left(\Theta-t\right)\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z}k_I\left(x,y,z,T\right)I_1^i\left(x,y,z,t\right)d\,z\,d\,y\,d\,x\,d\,t\,.$

The approximations with the second and higher orders of the concentration of radiation defects were calculated framework of the standard iterative procedure of the method of averaging functional corrections [11-13]. Framework the procedure to find the approximations with the *n*-th order of the concentrations of radiation defects, we replace in the right sides of equations (1*a*) and (2*a*) the required distributions of concentrations *I* (*x*,*y*,*z*,*t*), *V*(*x*,*y*,*z*,*t*), $c_I(x,y,z,t)$ and $c_V(x,y,z,t)$ by the sum of the average value of the *n*-th-order approximation and the *n*-1-th approximation of these functions, i.e. for the sum $\sim_{n\circ}+^{\circ}{}_{n-1}(x,y,z,t)$. After such a replacement, we obtain the relations for the second approximations of the required concentrations

$$\frac{\hat{o}\;I_2\left(x,y,z,t\right)}{\hat{o}\;t}=\frac{\hat{o}}{\hat{o}\;x}\left[D_I\left(x,y,z,T\right)\frac{\hat{o}\;I_1\left(x,y,z,t\right)}{\hat{o}\,x}\right]+\frac{\hat{o}}{\hat{o}\;y}\left[D_I\left(x,y,z,T\right)\frac{\hat{o}\;I_1\left(x,y,z,t\right)}{\hat{o}\,y}\right]+$$

$$+\frac{\hat{o}}{\hat{o}\;z}\left[D_I\left(x,y,z,T\right)\frac{\hat{o}\;I_1\left(x,y,z,t\right)}{\hat{o}\;z}\right]-k_{I,V}\left(x,y,z,T\right)\left[\sim_{1I}+I_1\left(x,y,z,t\right)\right]\left[\sim_{1V}+V_1\left(x,y,z,t\right)\right]-$$

$$-k_{I,I}\left(x,y,z,T\right)\left[\sim_{1I}+I_1\left(x,y,z,t\right)\right]^2+\Omega\frac{\partial}{\partial x}\left\{\frac{D_{IS}}{kT}\nabla_S\ll\left(x,y,z,t\right)\int\limits_0^{L_z}\left[\sim_{2I}+I_1\left(x,y,W,t\right)\right]d\,W\right\}+$$

$$+\Omega\frac{\partial}{\partial y}\left\{\frac{D_{IS}}{kT}\nabla_S\ll\left(x,y,z,t\right)\int\limits_0^{L_z}\left[\sim_{2I}+I_1\left(x,y,W,t\right)\right]d\,W\right\}\qquad(1d)$$

$$\frac{\hat{o}\;V_2\left(x,y,z,t\right)}{\hat{o}\;t}=\frac{\hat{o}}{\hat{o}\;x}\left[D_V\left(x,y,z,T\right)\frac{\hat{o}\;V_1\left(x,y,z,t\right)}{\hat{o}\,x}\right]+\frac{\hat{o}}{\hat{o}\;y}\left[D_V\left(x,y,z,T\right)\frac{\hat{o}\;V_1\left(x,y,z,t\right)}{\hat{o}\,y}\right]+$$

$$+\frac{\hat{o}}{\hat{o}\;z}\left[D_V\left(x,y,z,T\right)\frac{\hat{o}\;V_1\left(x,y,z,t\right)}{\hat{o}\;z}\right]-k_{I,V}\left(x,y,z,T\right)\left[\sim_{1I}+I_1\left(x,y,z,t\right)\right]\left[\sim_{1V}+V_1\left(x,y,z,t\right)\right]-$$

$$-k_{V,V}\left(x,y,z,T\right)\left[\sim_{1V}+V_1\left(x,y,z,t\right)\right]^2+\Omega\frac{\partial}{\partial x}\left\{\frac{D_{VS}}{kT}\nabla_S\ll\left(x,y,z,t\right)\int\limits_0^{L_z}\left[\sim_{2V}+V_1\left(x,y,W,t\right)\right]d\,W\right\}+$$

$$+\Omega\frac{\partial}{\partial y}\left\{\frac{D_{VS}}{kT}\nabla_S\ll\left(x,y,z,t\right)\int\limits_0^{L_z}\left[\sim_{2V}+V_1\left(x,y,W,t\right)\right]d\,W\right\}$$

$$\frac{\partial \Phi_{2I}(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_{1I}(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_{1I}(x,y,z,t)}{\partial y}\right]+$$

$$+\frac{\partial}{\partial z}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_{1I}(x,y,z,t)}{\partial z}\right]+k_{I,I}(x,y,z,T)I^2(x,y,z,t)+$$

$$+\Omega\frac{\partial}{\partial x}\left\{\frac{D_{\Phi_I S}}{kT}\nabla_S\mu(x,y,z,t)\int\limits_0^{L_z}\left[\alpha_{2\Phi_I}+\Phi_{1I}(x,y,W,t)\right]dW\right\}+k_I(x,y,z,T)I(x,y,z,t)+$$

$$+\Omega\frac{\partial}{\partial y}\left\{\frac{D_{\Phi_I S}}{kT}\nabla_S\mu(x,y,z,t)\int\limits_0^{L_z}\left[\alpha_{2\Phi_I}+\Phi_{1I}(x,y,W,t)\right]dW\right\}+f_{\Phi_I}(x,y,z)\delta(t) \quad (2d)$$

$$\frac{\partial \Phi_{2V}(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_{1V}(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_{1V}(x,y,z,t)}{\partial y}\right]+$$

$$+\frac{\partial}{\partial z}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_{1V}(x,y,z,t)}{\partial z}\right]+k_{V,V}(x,y,z,T)V^2(x,y,z,t)+$$

$$+\Omega\frac{\partial}{\partial x}\left\{\frac{D_{\Phi_I S}}{kT}\nabla_S\mu(x,y,z,t)\int\limits_0^{L_z}\left[\alpha_{2\Phi_V}+\Phi_{1V}(x,y,W,t)\right]dW\right\}+k_V(x,y,z,T)V(x,y,z,t)+$$

$$+\Omega\frac{\partial}{\partial y}\left\{\frac{D_{\Phi_V S}}{kT}\nabla_S\mu(x,y,z,t)\int\limits_0^{L_z}\left[\alpha_{2\Phi_V}+\Phi_{1V}(x,y,W,t)\right]dW\right\}+f_{\Phi_V}(x,y,z)\delta(t).$$

Integration of left and right sides of the above equations (1*d*) and (2*d*) gives a possibility to obtain relations for the second-order approximations of required concentrations in the required form

$$I_2(x,y,z,t)=\frac{\partial}{\partial x}\int\limits_0^t D_I(x,y,z,T)\frac{\partial I_1(x,y,z,\tau)}{\partial x}d\tau+\frac{\partial}{\partial y}\int\limits_0^t D_I(x,y,z,T)\frac{\partial I_1(x,y,z,\tau)}{\partial y}d\tau+$$

$$+\frac{\partial}{\partial z}\int\limits_0^t D_I(x,y,z,T)\frac{\partial I_1(x,y,z,\tau)}{\partial z}d\tau-\int\limits_0^t k_{I,I}(x,y,z,T)\left[\alpha_{2I}+I_1(x,y,z,\tau)\right]^2d\tau-$$

$$-\int\limits_0^t k_{I,V}(x,y,z,T)\left[\alpha_{2I}+I_1(x,y,z,\tau)\right]\left[\alpha_{2V}+V_1(x,y,z,\tau)\right]d\tau+\Omega\frac{\partial}{\partial x}\int\limits_0^t\nabla_S\mu(x,y,z,\tau)\times$$

$$\times\frac{D_{IS}}{kT}\int\limits_0^{L_z}\left[\alpha_{2I}+I_1(x,y,W,\tau)\right]dW\,d\tau+\Omega\frac{\partial}{\partial y}\int\limits_0^t\nabla_S\mu(x,y,z,\tau)\int\limits_0^{L_z}\left[\alpha_{2I}+I_1(x,y,W,\tau)\right]dW\times$$

$$\times\frac{D_{IS}}{kT}d\tau+f_I(x,y,z) \quad (1e)$$

$$V_2(x,y,z,t)=\frac{\partial}{\partial x}\int\limits_0^t D_V(x,y,z,T)\frac{\partial V_1(x,y,z,\tau)}{\partial x}d\tau+\frac{\partial}{\partial y}\int\limits_0^t D_V(x,y,z,T)\frac{\partial V_1(x,y,z,\tau)}{\partial y}d\tau+$$

$$+\frac{\hat{o}}{\hat{o}\ z}\int_0^t D_V(x,y,z,T)\frac{\hat{o}\ V_1(x,y,z,{}^2)}{\hat{o}\ z}d{}^2-\int_0^t k_{V,V}(x,y,z,T)\left[\sim_{2V}+V_1(x,y,z,{}^2)\right]^2 d{}^2-$$

$$-\int_0^t k_{I,V}(x,y,z,T)\left[\sim_{2I}+I_1(x,y,z,{}^2)\right]\left[\sim_{2V}+V_1(x,y,z,{}^2)\right]d{}^2+\Omega\frac{\partial}{\partial x}\int_0^t\nabla_S\ll(x,y,z,{}^2)\times$$

$$\times\frac{D_{VS}}{kT}\int_0^{L_z}\left[\sim_{2V}+V_1(x,y,W,{}^2)\right]dW\,d{}^2+\Omega\frac{\partial}{\partial y}\int_0^t\nabla_S\ll(x,y,z,{}^2)\int_0^{L_z}\left[\sim_{2V}+V_1(x,y,W,{}^2)\right]dW\times$$

$$\times\frac{D_{VS}}{kT}d{}^2+f_V(x,y,z)$$

$$\Phi_{2I}(x,y,z,t)=\frac{\hat{o}}{\hat{o}\ x}\int_0^t D_{\Phi_I}(x,y,z,T)\frac{\hat{o}\ \Phi_{1I}(x,y,z,{}^2)}{\hat{o}\ x}d{}^2+\frac{\hat{o}}{\hat{o}\ y}\int_0^t\frac{\hat{o}\ \Phi_{1I}(x,y,z,{}^2)}{\hat{o}\ y}\times$$

$$\times D_{\Phi_I}(x,y,z,T)\,d{}^2+\frac{\hat{o}}{\hat{o}\ z}\int_0^t D_{\Phi_I}(x,y,z,T)\frac{\hat{o}\ \Phi_{1I}(x,y,z,{}^2)}{\hat{o}\ z}d{}^2+\Omega\frac{\partial}{\partial x}\int_0^t\frac{D_{\Phi_I S}}{kT}\nabla_S\ll(x,y,z,{}^2)\times$$

$$\times\int_0^{L_z}\left[\sim_{2\Phi_I}+\Phi_{1I}(x,y,W,{}^2)\right]dW\,d{}^2+\Omega\frac{\partial}{\partial y}\int_0^t\nabla_S\ll(x,y,z,{}^2)\int_0^{L_z}\left[\sim_{2\Phi_I}+\Phi_{1I}(x,y,W,{}^2)\right]\times$$

$$\times\frac{D_{\Phi_I S}}{kT}dW\,d{}^2+\int_0^t k_{I,I}(x,y,z,T)I^2(x,y,z,{}^2)\,d{}^2+\int_0^t k_I(x,y,z,T)I(x,y,z,{}^2)\,d{}^2+$$

$$+f_{\Phi_I}(x,y,z)\qquad(1e)$$

$$\Phi_{2V}(x,y,z,t)=\frac{\hat{o}}{\hat{o}\ x}\int_0^t D_{\Phi_V}(x,y,z,T)\frac{\hat{o}\ \Phi_{1V}(x,y,z,{}^2)}{\hat{o}\ x}d{}^2+\frac{\hat{o}}{\hat{o}\ y}\int_0^t\frac{\hat{o}\ \Phi_{1V}(x,y,z,{}^2)}{\hat{o}\ y}\times$$

$$\times D_{\Phi_V}(x,y,z,T)\,d{}^2+\frac{\hat{o}}{\hat{o}\ z}\int_0^t D_{\Phi_V}(x,y,z,T)\frac{\hat{o}\ \Phi_{1V}(x,y,z,{}^2)}{\hat{o}\ z}d{}^2+\Omega\frac{\partial}{\partial x}\int_0^t\frac{D_{\Phi_V S}}{kT}\nabla_S\ll(x,y,z,{}^2)\times$$

$$\times\int_0^{L_z}\left[\sim_{2\Phi_V}+\Phi_{1V}(x,y,W,{}^2)\right]dW\,d{}^2+\Omega\frac{\partial}{\partial y}\int_0^t\nabla_S\ll(x,y,z,{}^2)\int_0^{L_z}\left[\sim_{2\Phi_V}+\Phi_{1V}(x,y,W,{}^2)\right]\times$$

$$\times\frac{D_{\Phi_V S}}{kT}dW\,d{}^2+\int_0^t k_{V,V}(x,y,z,T)V^2(x,y,z,{}^2)\,d{}^2+\int_0^t k_V(x,y,z,T)V(x,y,z,{}^2)\,d{}^2+$$

$$+f_{\Phi_V}(x,y,z).$$

Average values of second-order approximations of the considered functions were calculated by using relation, which is analogous to relation (9) (see, for example, [11-13]). Substitution of relations (1*e*) and (3*e*) into relation (9) gives a possibility to obtain following relations for the required average values $\sim_{2\circ}$

$$\sim_{2cI}=0,\ \sim_{2cV}=0,\ \sim_{2V}=\sqrt{\frac{(b_3+E)^2}{4b_4^2}-4\left(F+\frac{\Theta a_3F+\Theta^2L_xL_yL_zb_1}{b_4}\right)}-\frac{b_3+E}{4b_4},$$

$$\sim_{2I}=\frac{C_V-\sim_{2V}^2S_{VV00}-\sim_{2V}\left(2S_{VV01}+S_{IV10}+\Theta L_xL_yL_z\right)-S_{VV02}-S_{IV11}}{S_{IV01}+\sim_{2V}S_{IV00}},$$

Where $b_4 = \frac{1}{\Theta L_x L_y L_z} S_{IV00}^2 S_{VV00} - \frac{1}{\Theta L_x L_y L_z} S_{VV00}^2 S_{II00}$,

$$b_3 = -\left(2S_{VV01} + S_{IV10} + \Theta L_x L_y L_z\right)\times$$

$$\times \frac{S_{II00}S_{VV00}}{\Theta L_x L_y L_z} + \frac{S_{IV00}S_{VV00}}{\Theta L_x L_y L_z}\left(S_{IV01} + 2S_{II10} + S_{IV01} + \Theta L_x L_y L_z\right) + \frac{2S_{VV01} + S_{IV10} + \Theta L_x L_y L_z}{\Theta L_x L_y L_z} S_{IV00}^2 -$$

$$-\frac{S_{IV00}^2 S_{IV10}}{\Theta^3 L_x^3 L_y^3 L_z^3},$$

$$b_2 = \frac{S_{II00}S_{VV00}}{\Theta L_x L_y L_z}\left(S_{VV02} + S_{IV11} + C_V\right) - \left(\Theta L_x L_y L_z - 2S_{VV01} + S_{IV10}\right)^2 + \frac{S_{IV01}S_{VV00}}{\Theta L_x L_y L_z}\times$$

$$\times\left(\Theta L_x L_y L_z + 2S_{II10} + S_{IV01}\right) + \frac{\Theta L_x L_y L_z + S_{IV01} + 2S_{II10} + 2S_{IV01}}{\Theta L_x L_y L_z}\left(2S_{VV01} + \Theta L_x L_y L_z + S_{IV10}\right)\times$$

$$\times S_{IV00} - S_{IV00}^2\frac{C_V - S_{VV02} - S_{IV11}}{\Theta L_x L_y L_z} + \frac{C_I S_{IV00}^2}{\Theta^2 L_x^2 L_y^2 L_z^2} - 2\frac{S_{IV10}S_{IV00}S_{IV01}}{\Theta L_x L_y L_z},$$

$$b_1 = \left(S_{IV11} + S_{VV02} + C_V\right)\times$$

$$\times S_{II00}\frac{2S_{VV01} + S_{IV10} + \Theta L_x L_y L_z}{\Theta L_x L_y L_z} + S_{IV01}\frac{\Theta L_x L_y L_z + 2S_{II10} + S_{IV01}}{\Theta L_x L_y L_z}\left(2S_{VV01} + S_{IV10} + \Theta L_x L_y L_z\right) -$$

$$\times L_y L_z\big) - S_{IV00}\frac{C_V - S_{VV02} - S_{IV11}}{\Theta L_x L_y L_z}\left(3S_{IV01} + 2S_{II10} + \Theta L_x L_y L_z\right) + 2C_I S_{IV00}S_{IV01} - \frac{S_{IV10}S_{IV01}^2}{\Theta L_x L_y L_z}$$

,

$$b_0 = S_{II00}\frac{\left(S_{IV00} + S_{VV02}\right)^2}{\Theta L_x L_y L_z} - S_{IV01}\frac{C_V - S_{VV02} - S_{IV11}}{\Theta L_x L_y L_z}\left(\Theta L_x L_y L_z + 2S_{II10} + S_{IV01}\right) + 2C_I S_{IV01}^2 -$$

$$-S_{IV01}\frac{C_V - S_{VV02} - S_{IV11}}{\Theta L_x L_y L_z}\left(\Theta L_x L_y L_z + 2S_{II10} + S_{IV01}\right),\ C_I = \frac{\sim_{1I}\sim_{1V}}{\Theta L_x L_y L_z}S_{IV00} + \frac{\sim_{1I}^2 S_{II00}}{\Theta L_x L_y L_z} -$$

$$-\frac{S_{II20}S_{II20} + S_{IV11}}{\Theta L_x L_y L_z},\ C_V = \sim_{1I}\sim_{1V}S_{IV00} + \sim_{1V}^2 S_{VV00} - S_{VV02} - S_{IV11},$$

$$E = \sqrt{8y + \Theta^2\frac{a_3^2}{a_4^2} - 4\Theta\frac{a_2}{a_4}},$$

$$F = \frac{\Theta a_2}{6a_4} + \sqrt[3]{\sqrt{r^2 + s^3} - r} - \sqrt[3]{\sqrt{r^2 + s^3} + r},$$

$$r = \frac{\Theta^3 b_2}{24b_4^2}\left(4b_0 - \Theta L_x L_y L_z\frac{b_1 b_3}{b_4}\right) - b_0\frac{\Theta^2}{8b_4^2}\times$$

$$\times\left(4\Theta b_2-\Theta^2\frac{b_3^2}{b_4}\right)-\frac{\Theta^3 b_2^3}{54 b_4^3}-L_x^2L_y^2L_z^2\frac{\Theta^4 b_1^2}{8 b_4^2},\ s=\Theta^2\frac{4b_0b_4-\Theta L_xL_yL_zb_1b_3}{12b_4^2}-\frac{\Theta b_2}{18b_4}.$$

In this work, the required distributions of temperature and concentration of radiation defects, as well as the components of the displacement vector were calculated by using the second-order approximations framework the method of averaging functional corrections. The approximation is usually enough good approximation to make qualitative analysis and to obtain some quantitative results. All obtained results have been checked by comparison with results of numerical simulations.

DISCUSSION

In this section we analyzed relaxation of mismatch-induced stresses and spatio-temporal distributions of radiation defects. It was obtained, that it is practicably to choose energy of the implanted ions so, that maximum distribution of radiation defects (see Fig. 1) would be inside of the heterostructure after the interface between its layers. Taking into account that the value of the diffusion coefficient of interstitial atoms is usually larger, than value of diffusion coefficient of vacancies, with time, in the neighborhood of interface between the heterostructure layers, quantity of radiation vacancies will larger, than quantity of interstitial atoms. In this case, at £$_0$>0, the value of mismatch-induced stresses decreases with a simultaneous decreasing of quantity of radiation vacancies. The dependences of the values of the displacement vector and the concentration of vacancies on the coordinate, which is perpendicular to the interface between the heterostructure layers, are shown on Figs. 2 and 3 respectively. Probably, the decreasing of quantity of vacancies became more compact in the epitaxial layer under influence of mismatch-induced stresses. At the same time with increasing of density of the epitaxial layer decreasing of value of mismatch-induced stresses could be obtained, probably, by increasing of the density of atoms of the epitaxial layer. Analysis of distribution of temperature shows that it is advisable to choose the frequency of electromagnetic radiation so, that the thickness of the skin layer would be comparable to the width of the irradiated area by radiation processing. Decreasing of the considered frequency is not attracted an interest, since the energy of the electromagnetic field will be spent on the area of the heterostructure, which was not damaged during radiation processing. Increasing of frequency leads to the fact that the deepest area damaged during radiation treatment will not be annealed.

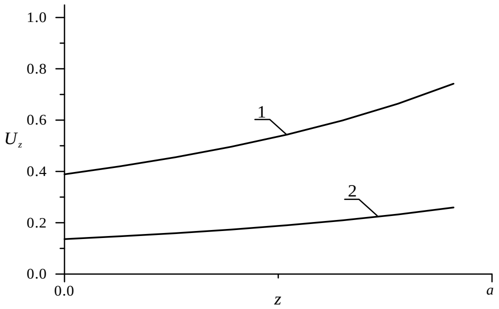

Fig. 2. Normalized dependences of the component of the displacement vector u_z on the z coordinate for epitaxial layers before radiation processing (curve 1) and after radiation processing (curve 2)

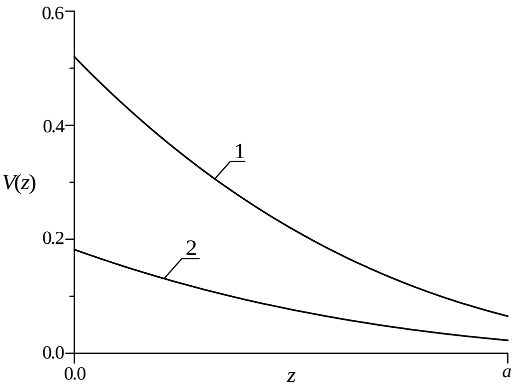

Fig. 3. Normalized dependences of vacancy concentrations on the z coordinate in unstressed (curve 1) and stressed (curve 2) unprocessed materials

CONCLUSION

In this paper, we consider the effect of microwave annealing of radiation defects generated during radiation processing of a heterostructure with account relaxation of mismatch-induced stresses. Based on the analysis we formulate recommendations to decrease the considered stresses. An analytical approach for analyzing of mass and heat transfer is introduced. The approach gives a possibility to take into account changes of parameters of processes in space and time, as well as the nonlinearity of the considered processes.

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